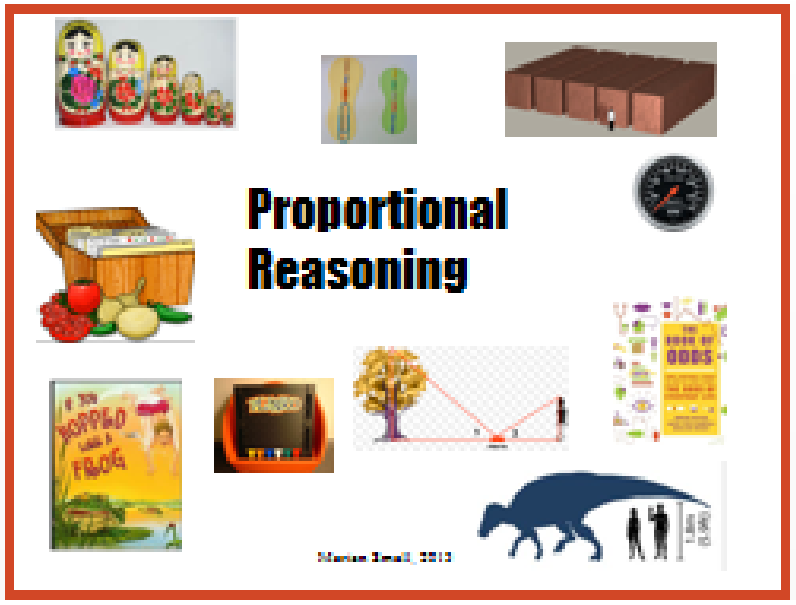
Proportional Reasoning



Marian Small, 2013

I wonder!

- **How long will it take you before you've turned on a light 100 times?**

Proportional reasoning

- **deliberate use of multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another.**

What does the light problem have to do

- **with proportional reasoning?**

Another term

- ***multiplicative thinking***

Why is this topic important?

- **useful in real life**
- **in so many places in math curriculum from K – 12**

Even more real-life examples

- **Exchanging coins**
- **Exchanging currency**

Even more real-life examples

- **Changing measurement units, e.g. centimetres to inches**
- **Choosing a best buy in terms of money or fuel efficiency or…**

Even more real-life examples

- **Calculating time based on speed**
- **Growth rates for children/ percentiles**

Even more real-life examples

- **Drawing scale diagrams to decorate**
- **Xerox reductions …..**

In the curriculum

Most obvious spots

- **Grades 4 – 8: sections under number entitled Proportional Reasoning**
- **Grade 9: expectations related to problems arising from applications of percent, ratio, rate and proportion**

But there are SO MANY more

Examples

- **Grade 1- 2: skip counting**
- **Grade 1 on: any measuring activity using units**

But there are SO MANY more

Examples

- **Grade 2 on: place value work**
- **Grade 2- 5: coin/money exchanges**

But there are SO MANY more

Examples

- **Grade 2: tally charts**
- **Grade 3 on: multiplication/division**

But there are SO MANY more

Examples

- **Grade 3 on: fractions**
- **Grade 3 on: graphs with scales**

But there are SO MANY more

Examples

- **Grade 3 on: probability**
- **Grade 4 on: solving problems relating to magnitudes of 1000, etc.**

But there are SO MANY more

Examples

- **Grade 4 on: area/volume formulas**
- **Grade 4 on: unit conversions**
- **Grade 4 on: linear patterns**

But there are SO MANY more

Examples

- **Grade 5 on: mean of a set of data**
- **Grade 6 on: pattern rules involving linear patterns**

But there are SO MANY more

Examples

- **Grade 6 on: percent work**
- **Grade 6: rotation work with patterns**

But there are SO MANY more

Examples

- **Grade 7 on: similarity, dilatations**
- **Grade 7 on: solving linear equations by, for example, multiplying both sides by the same amount**

But there are SO MANY more

Examples

- **Grade 8 - 10: linear relationships**
- **Grade 10: midpoint formula**
- **Grade 10 on: trigonometry**

But there are SO MANY more

Examples

- **Grade 11: arithmetic sequences**
- **Grade 11: simple interest**
- **Grade 11: certain aspects of transformations of functions**

But there are SO MANY more

Examples

- **Grade 11 workplace: unit rates, taxes, discounts, etc.**
- **Grade 12: average rate of change**

What about

- **the images on the cover slide?**
- **Choose one of those images and think about how it relates to proportional reasoning.**

Stages of proportional reasoning

Problem: If 24 items are to be distributed to Ann and Ben in a ratio of 3:5, how many does each person get?

First solve it (with a partner) your own way.

Stages of proportional reasoning

Intuitive:

- **Think half and half (12 and 12)**
- **Realize it's wrong.**
- **So try something else, e.g. 8 and 16.**
- **Realize it's wrong, but close.**
- **So try 9 and 15.**

Stages of proportional reasoning

Additive:

Take 8 from the pile of 24 and distribute 3 to Ann and 5 to Ben and repeat until the pile is gone.

Stages of proportional reasoning

Equivalent ratio:

Realize that 3:5 is equivalent to 9: 15 and 9 + 15 = 24, so Ann gets 9 and Ben gets 15.

Stages of proportional reasoning

Finding the unit:

Think of 24 as the whole. Divide the whole into 3 subunits of size 8 and conclude that in every subunit, 3 belong to one person and 5 the other. Since there are 3 subunits, the numbers each gets are 3x 3 and 3x5.

Stages of proportional reasoning

- **Determining the part from the whole: Take 3/8 of 24 and 5/8 of 24.**

Stages of proportional reasoning

- **Proportion:**

$$\frac{3}{x}=\frac{5}{24-x}$$

x is what Ann gets and 24 – x is what Ben gets.

72 – 3x = 5x and x = 9

Big Idea

- **It is often useful to think of one amount as how many of another amount.**
- **e.g. one loonie as 4 quarters**
- **1 week as 7 days**
- **1 wall as 5 panels**
- **300 m as 3 sets of 100m**
- **20 eggs as 1 2/3 dozen**

Related important ideas

- **If you use a bigger unit, you need fewer of them.**

Related important ideas

- **If units are close in size, you need about the same number of one as another to describe an amount.**

Related important ideas

- **If units are related, you can use that relationship to predict how many of one unit if you know how many of the other.**

Comparing changes

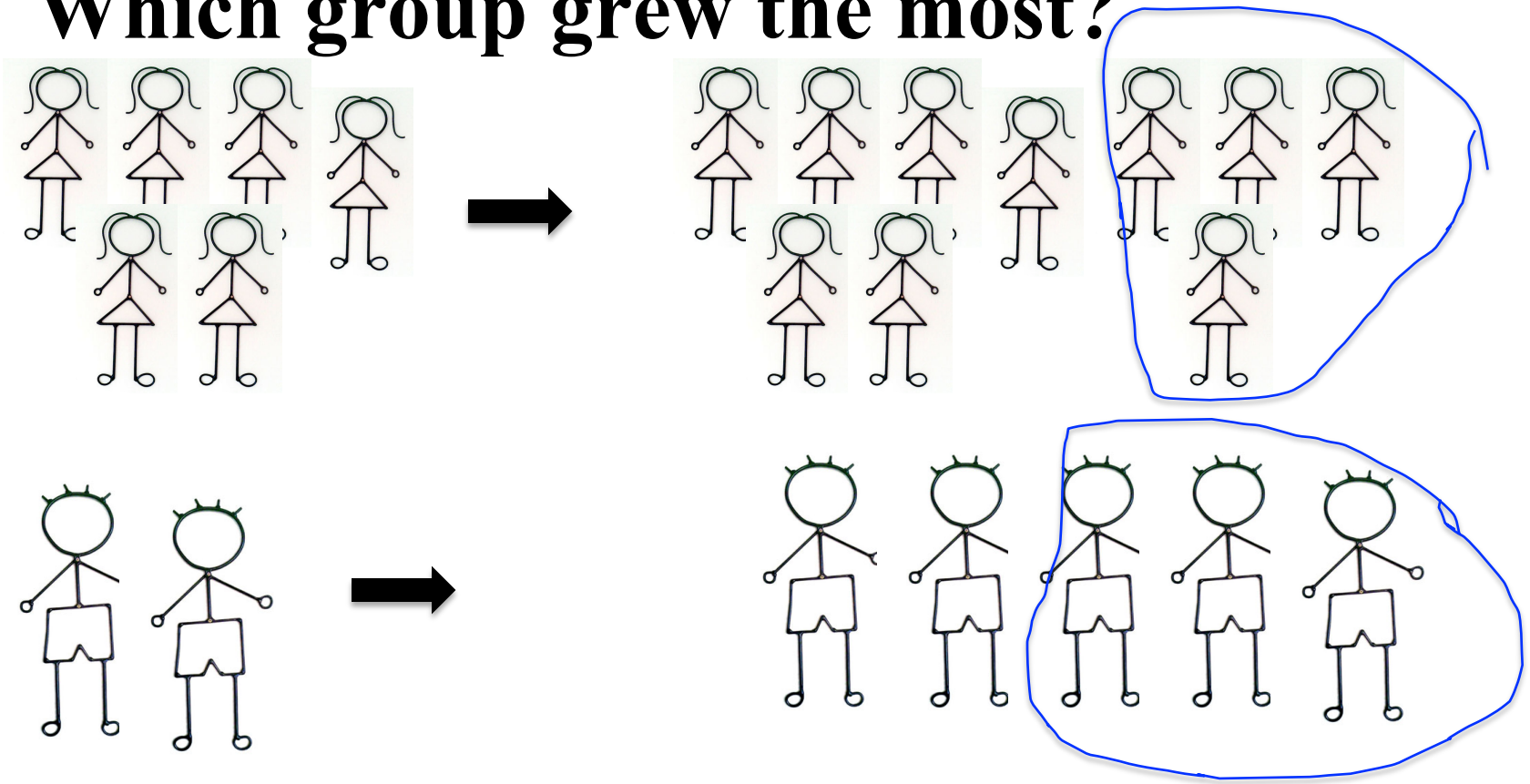
Which price changed the most?

$46 945.00 to $44 999.00

$5.99 to $2.99

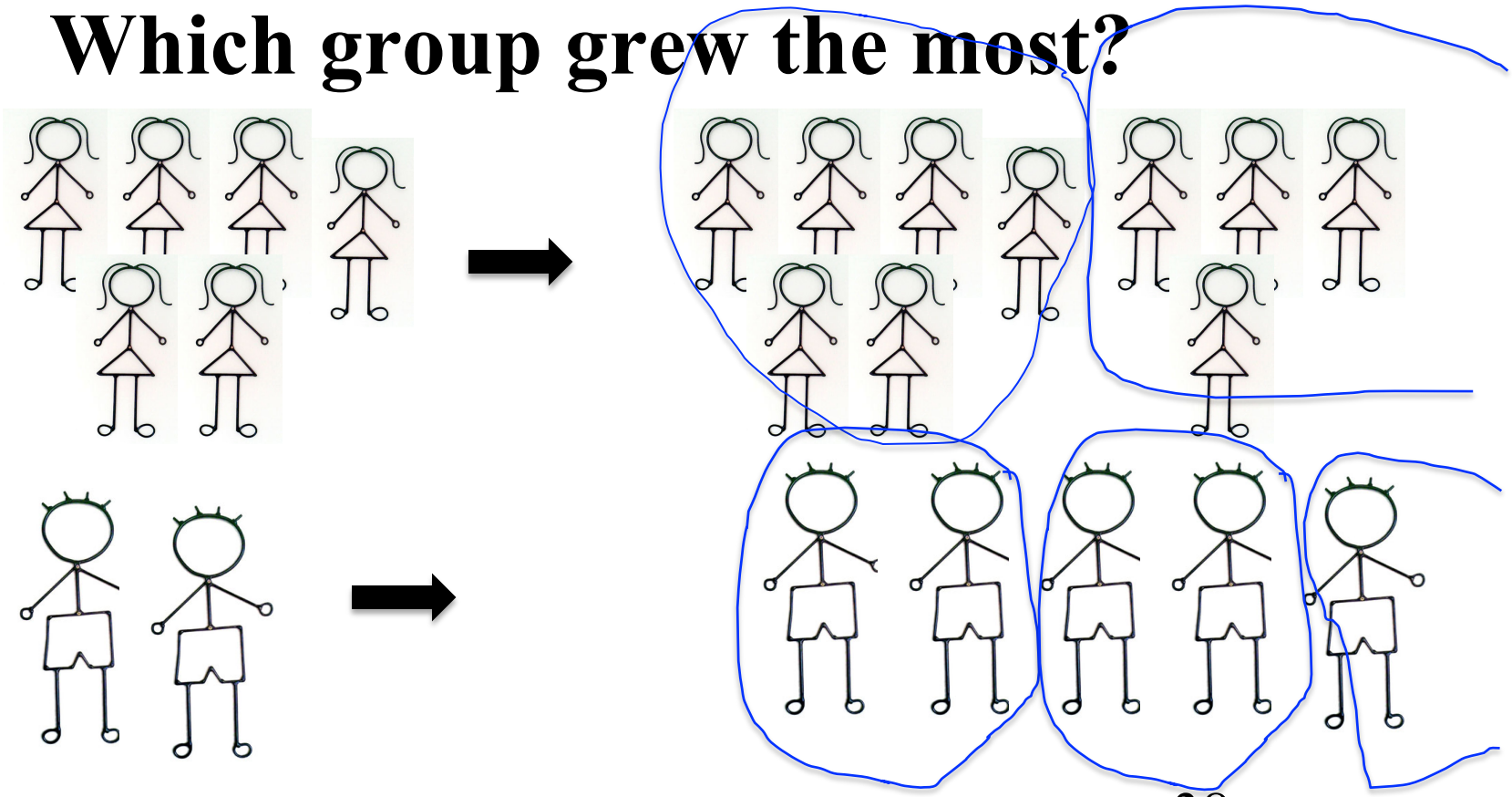
Primary version

Which group grew the most?



Primary version

Which group grew the most?



Related important ideas

- **Any number can be compared to any other number multiplicatively, e.g. 9 can be compared to 2 by thinking of it as 4 ½ twos.**

  And 2 can be compared to 9 as 2/9 of a 9.

Related important ideas

- **Two numbers can be far apart from an additive point of view but not from a multiplicative point of view, e.g. 1000 and 100 are 900 apart, but 1000 is only 10 100s.**
- **Or vice versa, 10 and 6 are 4 apart additively, but both are between 1 and 2 6s.**

Related important ideas

- **Using a fraction, decimal or percent is a way of comparing numbers multiplicatively.**
- **For example, 2/3 tells us that 2 is only 2/3 of a 3.**
- **0.4 is a way to compare 4 to 10**
- **35% is a way to compare 35 to 100**

Let's look at the types of

- **problems students might solve that involve proportional reasoning.**
- **As we move forward, think about how critical representation will be in students making sense of the tasks.**

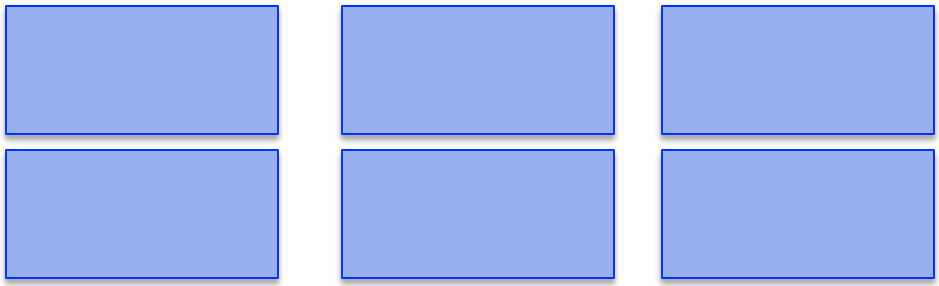
Costs

$15 for 6 boxes

$?? for 4 boxes

Ways to solve

- **2 boxes cost $5, so 4 boxes cost $10**



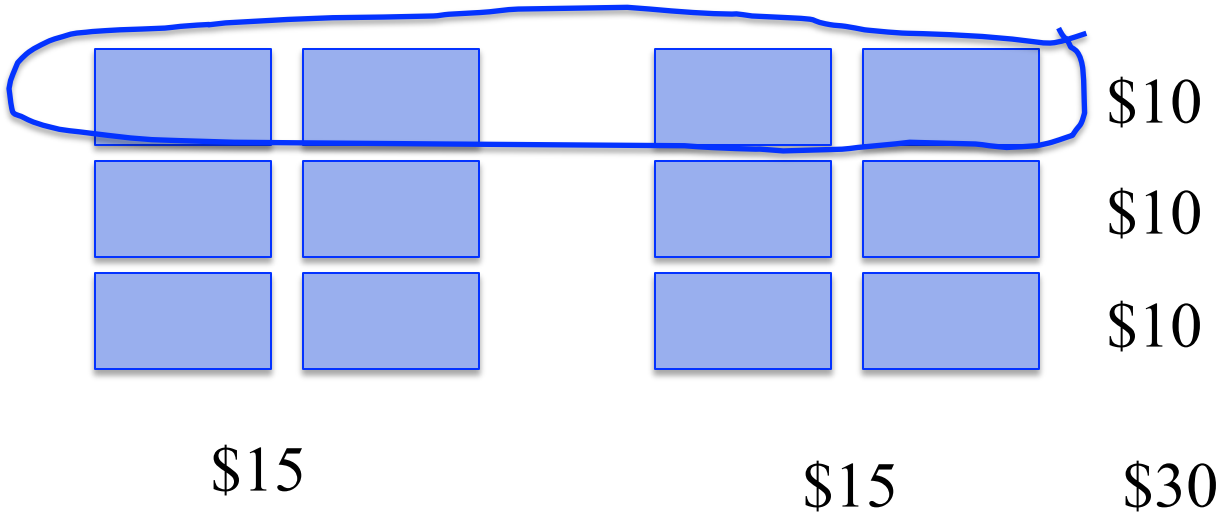
$5 $5 $5

Ways to solve

- **1 box costs $2.50 (15 ÷ 6),**
  so 4 boxes cost $10 (4 x 2.50)

Ways to solve

- **12 boxes cost $30, so 4 boxes cost $10**



Dogs

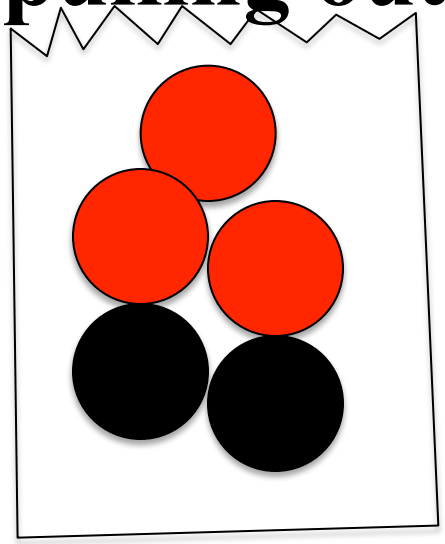
- **1 out of every 3 Canadian households has a dog.**
- **About how many dogs would you predict for the students in your class?**
- **How might a student represent this?**

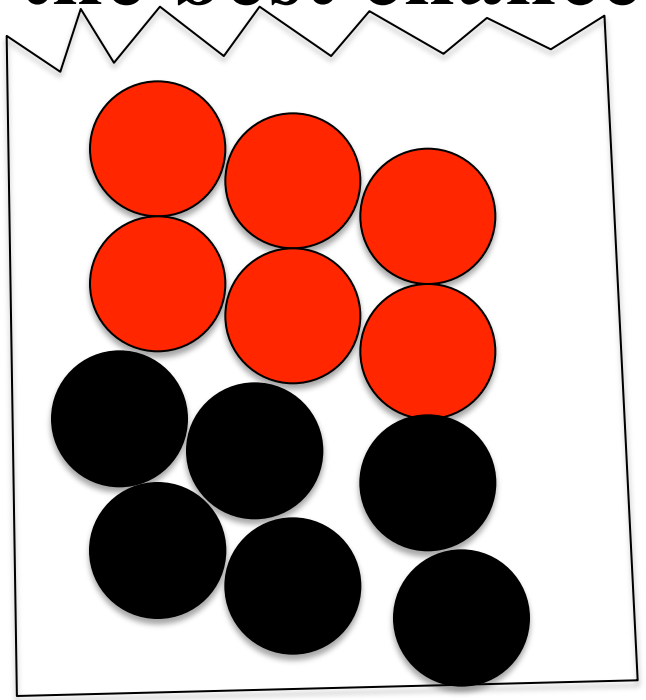
More advanced version

- **On average, Canadians consume 18% of their daily calories at breakfast.**
- **Is that true in your class?**

Probability

- **You are pulling out a counter from each bag.**
- **Which bag gives you the best chance of pulling out a red?**





Speeds

- **A car goes 280 km in 3 hours.**
- **How far, at that speed, will they go in another 1.5 hours?**

Fraction comparisons

- **How do you know that $\frac{18}{37}$ is a bit less than $\frac{1}{2}$?**

Length

- **How long is a line of 1 000 000 pennies?**

Printer time

- http://mrmeyer.com/threeacts/printjob/

Getting to 1000

- **Which sequence will get to 1000 first?**

15, 25, 35, 45,….

500, 502, 504,…

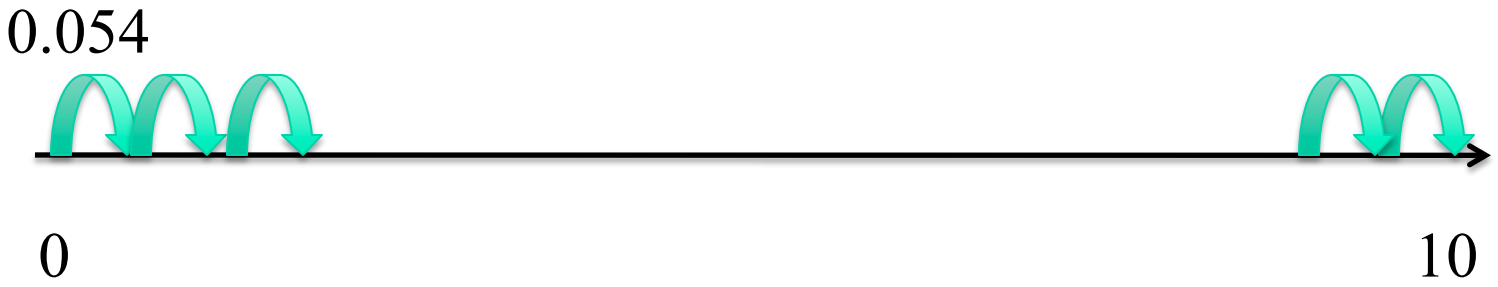
Changing measurements

- **A container holds [] L .**
- **If you measured how much it holds in quarts, it would be [] + 10 quarts.**

- **How might a student represent this problem to help him/her solve it?**

Changing measurements

- **1 quart = 0.946 L**
- **1 – 0.946 = 0.054 so**

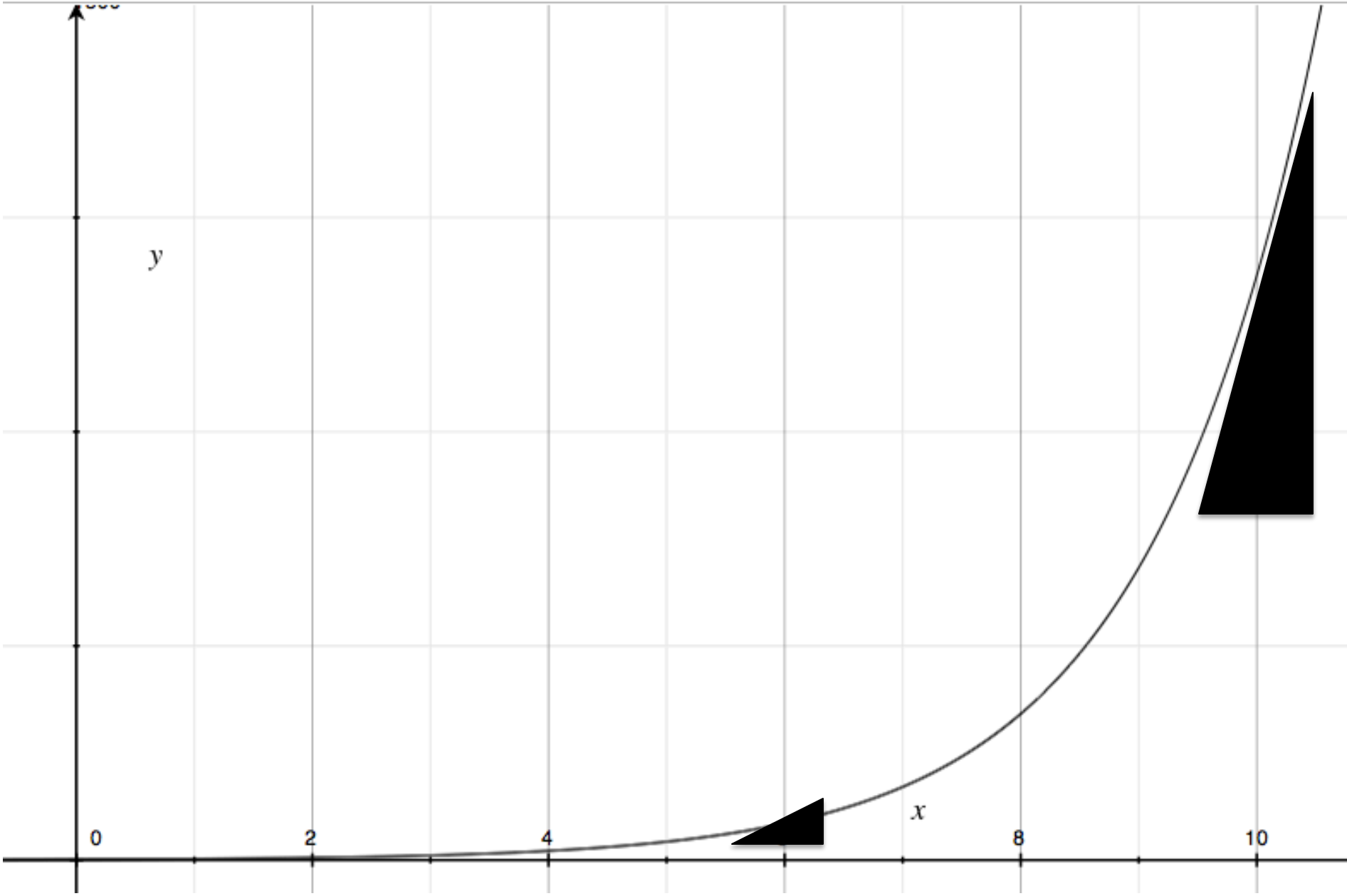


How much faster?

- **You normally drive 90 km/h on a certain road. How much faster would you have to go to save 15 minutes on a 400 km trip on that road?**

Functions

- **How much faster is the function shown on the next slide growing near $x = 10$ than near $x = 6$?**


y
x
0
2
4
8
10

Estimation

A Fermi problem, e.g.

Estimate the number of square centimetres of pizza that all of the students in Ottawa eat in one week.

What can you do to build proportional reasoning?

- **Many suitable tasks are suggested as models.**

You could ask

- **You can represent a certain amount of money with exactly 6 identical coins.**
- **How much could it be worth?**

And continue by asking

- **How did I know that nobody would say $1.00?**
- **Why are some amounts you said so much more than other amounts?**
- **Why is the number you said even?**

You could ask

- **You can arrange a batch of ABOUT 50 counters into equal groups. How many groups and of what size might they be?**

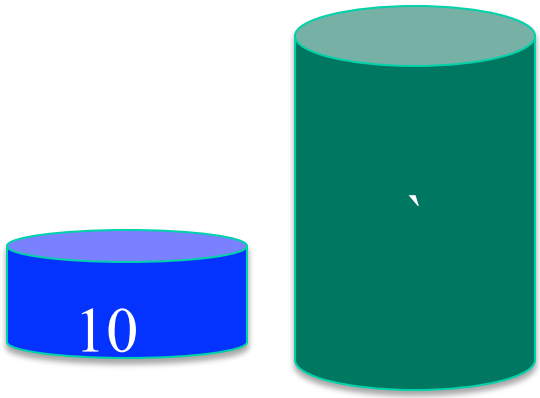
Follow up by asking

- **What was the biggest group size anyone had? Why?**
- **When did someone have a lot of groups?**
- **When did someone have a big group size?**
- **When could there be 2 groups?**

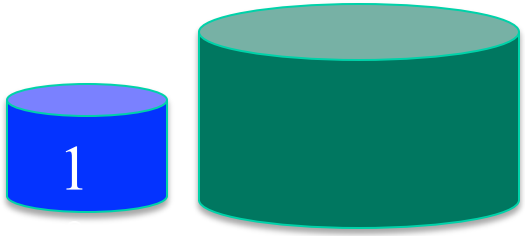
You could ask:

How many marbles do you think the big container could hold?

Choice 1:



Choice 2:



Common questions:

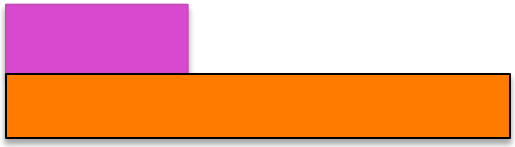
- **Are there more than 10 marbles in the big container? How do you know?**
- **Do you think there are more than 20 marbles? Why or why not?**
- **Did it matter how wide the dark blue container (with 10 marbles) was?**
- **How?**

Common questions:

- **Did it matter how high the dark blue container of 10 was?**
- **How?**
- **How did you decide how many marbles?**
- **What if there had only been 5 marbles in the small can? How would your answer change?**

You could ask

- **How many pink rods would it take to measure the table?**



You could ask

- **How many nickels would it take to have 75¢?**

You could ask

- **How many ears would I draw if I draw 8 cows?**
- **How many legs?**

You could ask

- **How many numbers would I need to write (say) to continue this way to get to 50?**

  12, 14, 16, 18, 20,….

You could ask

- **You can show an amount of cookies exactly using groups of 6 cookies.**
- **How do you know that you can also show it exactly using groups of 3 cookies?**
- **What about using groups of 4 cookies?**

You could

regularly use multiplicative language such as:

- **Twice as much**
- **Four times as big**
- **Half as many**
- **Two thirds as heavy**

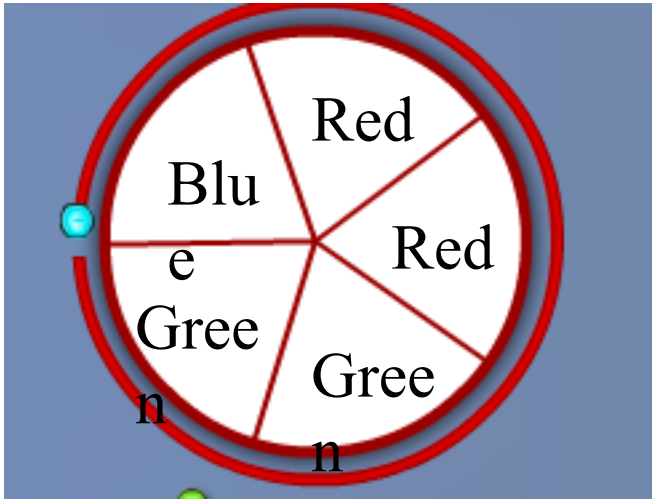
You could ask

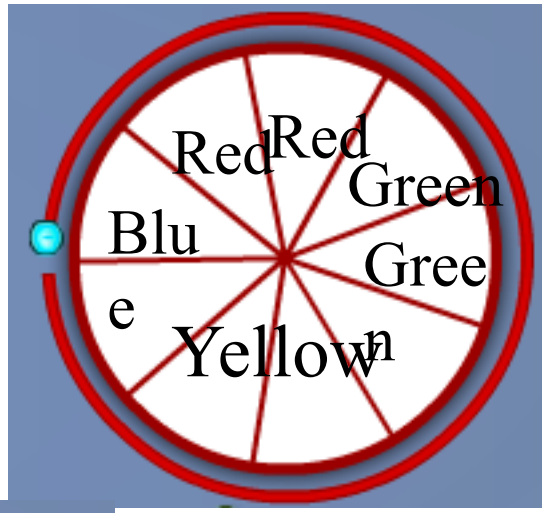
- **I read three times as many books as my sister.**
- **How many might we each have read?**
- **Do you think I could have read exactly 20 books?**

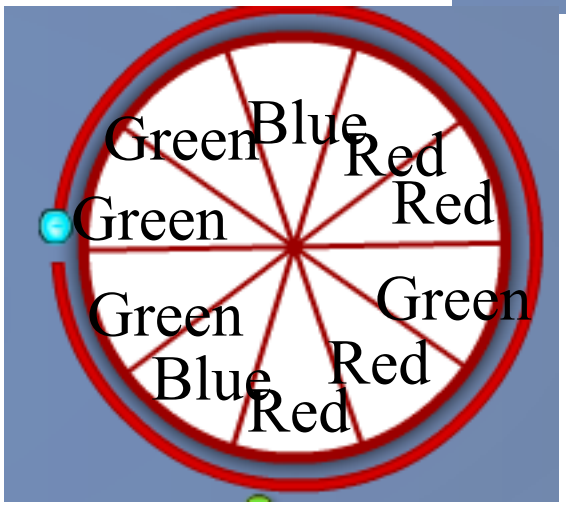
A Colourful Spinner

- **I spin a spinner.**
- **I am twice as likely to get red as blue.**
- **I am half as likely to get blue as green.**
- **What could the probability of green be?**

Possibilities







Examples

- **Create a sentence that uses the words double, triple and the numbers 3 and 8.**

Examples

For example:

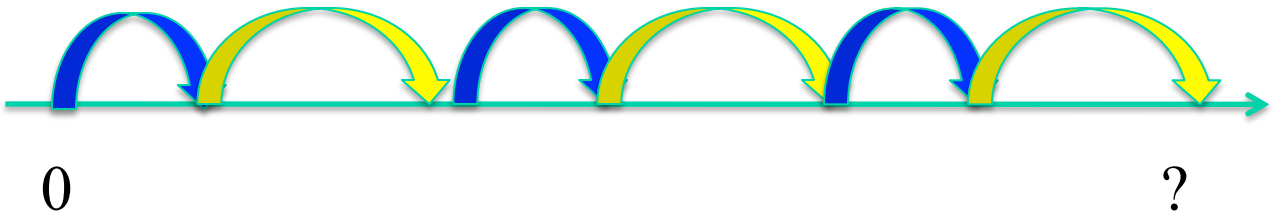
- **If you double 3 and add triple 8, you get 30.**
- **Triple 3 is less than double 8.**
- **We ordered 8 triple scoops and 3 double scoops for the team.**

You could ask

You know that the yellow arrow is a little longer than the blue one.

Both are whole number amounts.

What could ? be? How do you know?



You could ask

- **A sentence has 40 letters in it. What number of words do you think it probably has? Why?**

You could ask

- **Sam has 50 dimes.**
- **If she traded for nickels, how many would she get?**
- **If she traded for quarters, how many would she get?**
- **How could you show this?**

You could ask

- **About how many ceiling tiles are there in the whole school?**

You could ask

- **A number that can be written in the form a^b is about 2/3 of a number that can be written in the form c^d. What could a, b, c and d be?**

You could ask

- **You draw a scale diagram and a ___ m distance is represented as ____ cm.**
- **Choose values for the blanks.**
- **Then describe how a 17 m and 3.2 m distance would be represented.**

You could ask

- **A function grows at about the same rate as f(x) = 5x – 100. What could it be? Explain.**

You could ask

- **When *x* increases by 20,**
  y decreases by 3.
- **What equation could relate *x* to *y*?**

Notice

- **To get from the pattern**
- **2, 4, 6, 8, 10,… to**
- **5, 10, 15, 20, 25,..**
- **You can take half and multiply by 5.**

Notice

- **To get from the pattern**
- **2, 4, 6, 8, 10,… to**
- **11, 21, 31, 41, 51..**
- **You can take half and multiply by 10 and add 1.**

You could ask

There are two linear patterns.

To get from pattern 1 to pattern 2:

- **Subtract 1**
- **Take half**
- **Multiply by 3**
- **Add 5.** **What are they?**

You could ask

- **The perimeter of one square is 1/3 as long as the perimeter of another. What do you know about the side lengths?**
- **How could you represent this?**

You could ask

- **Jane is 8. Her mom is 38.**
- **When did or will her mom be twice as old as Jane?**
- **When would it happen again (or would it)?**

We are going to work on problems

- **You will have a choice of problems, depending on the level of interest to you.**
- **You will solve two problems with others and be sure to record your thinking.**

We will then talk about:

- **what important ideas about proportional reasoning are being addressed**
- **what specific curriculum expectations are being addressed**

We will then talk about:

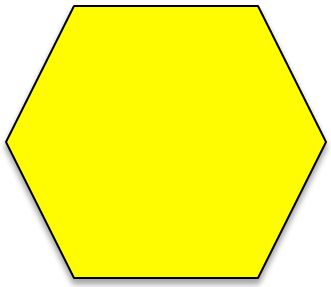
- **what to listen for in solutions (success criteria) and how that impacts what you do next**
- **how the sophistication of proportional reasoning grows over the grades**

Problem 1

- **You have more than 10 counters.**
- **When you put them in groups of 3, 1 counter is left over.**
- **When you put them in groups of 4, 3 counters are left over.**
- **How many counters might you have?**
- **Are there more groups of 3 or 4? Why?**

Problem 2

- **How many hexagon pattern blocks would it take to cover a 1 m x 1m space?**



Problem 3

- **Connor had 3/5 as much money saved as Aidan.**
- **Once Aidan gave Connor $9, Connor had almost 4/5 as much as Aidan.**
- **How much did each start with?**

Problem 4

- **Ben drove a total of 483 km.**
- **For part of that distance, he averaged 97 mph but for part of it he averaged only 64 mph.**
- **If the whole trip took 5 hours and 50 minutes, how much of it was at 97 km/h and how much of it was at 64 km/h?**

Problem 5

- **An area is 10 fewer square metres than square yards.**
- **What is the area in square metres?**

Let's share solutions

- **As you listen, think about what the responses told you about solvers' understandings.**
- **Also consider the feedback you would give them.**

Did you notice that…

- **Some of the same proportional reasoning ideas occur at earlier years as in later years.**
- **But there is growth in the content being brought to bear.**

Ministry resources

- **http://www.edu.gov.on.ca/eng/teachers/studentsuccess/ProportionReason.pdf**
- **http://resources.curriculum.org/secretariat/engagingmath/files/EngagingMathGuide.pdf**

Ministry resources

- **K-12: http://www.edugains.ca/resources/LearningMaterials/ContinuumConnection/BigIdeasQuestioning_ProportionalReasoning.pdf**

Ministry resources

- **Gr 7 – 12 http://www.edugains.ca/resources/LearningMaterials/ContinuumConnection/ProportionalReasoning.pdf**

Ministry resources

- **Math camppp materials on proportional reasoning**
- **http://gains-camppp.wikispaces.com/CAMPPP+2010**
- **http://gains-camppp.wikispaces.com/CAMPPP+2011+Home**

A new EQAO video

- **http://www.youtube.com/watch?v=LPkQvN3r8js**

Choose an expectation

- **Choose one of the expectations on the next slide.**
- **Create a meaningful proportional reasoning task.**
- **Think about the criteria you would use to determine if students have been successful.**

Expectations

- **Describe, thru investigation … relationship between the size of a unit of area and the number of units needed to cover …**
- **Compare fractions to …. 0, ½ and 1**
- **Predict frequency of outcome… using theoretical probability**
- **Develop formulas for volumes of…..**

Message

- **Proportional reasoning is about unitizing, grouping and counting groups, thinking of comparisons multiplicatively.**
- **This happens if you model it, talk about it, present tasks that allow for it, and encourage it.**
- **Being informal for a while may be useful.**

Project directions

- **PLCs focused on what proportional reasoning ideas to bring out from the curriculum**
- **Creating short videos or webinars to focus on lesson consolidation**

Project directions

- **Co-planning, co-teaching, collaborative inquiry using proportional reasoning focused lessons**
- **Creating short videos or webinars for teachers to help them better understand proportional reasoning ideas**

Project directions

- **Creating tasks for home, maybe some to be done with parents and some not**
- **Developing rich tasks (maybe digitally based or maybe not) for proportional reasoning, but always focused on.. "and this is to bring out…."**

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